The q-superoscillators in supersymmetric quantum systems and their q-supercoherent states

1994 J. Phys. A: Math. Gen. 27 5995

(http://iopscience.iop.org/0305-4470/27/17/030)

J. Phys. A: Math. Gen. 27 (1994) 5995–6004. Printed in the UK

The q-superoscillators in supersymmetric quantum systems and their q-supercoherent states

San-Ru Hao†, Guang-Hua Li‡ and Jun-Yan Long‡

† CCAST (World Laboratory), PO Box 8730, Beijing 100080, People's Republic of China and Calculating Physics Division, Department of Computer Teaching, Hunan Normal University, Hunan 410081, People's Republic of China§

‡ Department of Physics, Changsha Normal University of Water Resources and Electric Power, Hunan 410077, People's Republic of China

Received 3 June 1993, in final form 6 June 1994

Abstract. In this paper, the q-deformed quantum harmonic oscillators in the supersymmetric quantum systems are introduced. The q-supercoherent states associated with the q-deformed supersymmetric quantum harmonic oscillators are constructed explicitly, and their properties are investigated in detail. It is shown that the q-supercoherent states are completeness. The uncertainty relations for q-supercoherent states are discussed as well.

1. Introduction

It is well known that the usual coherent states [1–4] of Lie (super)algebras have wide applications to various branches of physics. Over the past few years, quantum groups and their representations [5, 6] have drawn considerable attention from mathematicians and physicists. A problem of interest is the consideration of coherent states associated with the quantum groups, called q-coherent states. Recently, q-coherent states of the q-harmonic oscillators [7–10] have been investigated by several authors, but the q-supercoherent states of the q-superoscillators in a supersymmetric quantum system have not been considered. Here we will investigate them in detail.

2. q-deformed superoscillators model

Let us consider the standard Witten supersymmetrization procedure [11] in $N = 2$ symmetric quantum mechanics. The two Hermitian (odd) supercharges Q_1 and Q_2, with the supersymmetric (even) Hamiltonian H, generate the well known (Lie) algebra sqm(2) characterized by the structure relations:

$$Q_1^2 = Q_2^2 = H \qquad \{Q_1, Q_2\} = 0 \qquad [Q_1, H] = [Q_2, H] = 0. \tag{2.1}$$

These algebra elements have the following forms in the one-dimensional spatial context [12, 13]:

$$Q_1 = (\tfrac{1}{2})^{1/2}(Mp_x + NW'(x)) \tag{2.2a}$$

$$Q_2 = (\tfrac{1}{2})^{1/2}(Np_x - MW'(x)) \tag{2.2b}$$

§ Mailing address.

where M, N are the Pauli matrix sigma-1 and sigma-2 respectively, and $W(x)$ is the so-called superpotential, while $W'(x)$ is the partial derivative of $W(x)$ with respect to x and p_x is the momentum operator as usual. The supersymmetric Hamiltonians can be described by the following:

$$H = \tfrac{1}{2}(p_x^2 + W'^2 + TW'') \tag{2.3}$$

where T is the Pauli matrix sigma-3.

Let us restrict ourselves here to harmonic oscillator-like systems by dealing with the superpotential given in [12]:

$$W(x) = \tfrac{1}{2}\omega x^2 \tag{2.4}$$

where ω is the corresponding angular frequency. The supersymmetric Hamiltonian H of the quantum harmonic oscillators then becomes

$$H = \tfrac{1}{2}(p_x^2 + \omega^2 x^2 + \omega T). \tag{2.5}$$

Then, let us observe the relation between the above supersymmetric quantum harmonic oscillator systems and the usual ones. We find that there is one term more than the usual number. This term comes from the supersymmetry.

We now define an annihilation operator (a) and a creation operator (a^+), respectively:

$$a = (2\omega)^{-1/2}(\omega x + \mathrm{i}p_x) \tag{2.6a}$$

$$a^+ = (2\omega)^{-1/2}(\omega x - p_x). \tag{2.6b}$$

From equations (2.6), it follows that

$$\begin{aligned} &[a, a^+] = 1 \\ &H = \tfrac{1}{2}\omega\{a^+, a\} + \tfrac{1}{2}\omega T. \end{aligned} \tag{2.7}$$

If we express H as components,

$$H_\pm = \tfrac{1}{2}\omega\{a^+, a\} \pm \omega/2 \tag{2.8}$$

the action of operator H on the eigenstate $\Psi_{n,m}$ may be written as

$$H\Psi_{n,m} = (\omega/2)(\{a^+, a\} + [f^+, f])\Psi_{n,m} \qquad m = 0, 1 \tag{2.9}$$

where f^+ and f are the fermionic creation operator and fermionic annihilation operator, respectively, while a and a^+ are the bosonic annihilation and creation operators defined by equations (2.6), respectively.

Making use of the eigenvalue equation (2.9), one can check easily that the eigenstates $\Psi_{n,m}$ may be expressed as

$$\Psi_{n,m} \equiv \|n, m\rangle = \frac{(a^+)^n (f^+)^m \|0\rangle}{\sqrt{n!m!}} \qquad m = 0, 1. \tag{2.10}$$

The action of operators $a(a^+)$ and $f(f^+)$ on the eigenstate $\Psi_{n,m}$ may be written as

$$a\|n,m\rangle = \sqrt{n}\|n-1,m\rangle \qquad a^+\|n,m\rangle = \sqrt{n+1}\|n+1,m\rangle \tag{2.11}$$

$$f\|n,m\rangle = \sqrt{m}\|n,m-1\rangle \qquad f^+\|n,m\rangle = \sqrt{m+1}\delta_{m1}\|n,m+1\rangle. \tag{2.12}$$

We now consider the q-deformation of the supersymmetric quantum harmonic oscillators (2.9), which are discussed in [14], and are different from the b-deformation defined in [12]. The Hamiltonian of the q-deformed superoscillators may be written as

$$H_q = (\omega/2)(\{a_q, a_q^+\} + [f_q^+, f_q]) \tag{2.13}$$

where a_q and a_q^+ are the q-deformed bosonic operators which satisfy the commutation relations [14]

$$[a_q, a_q^+] = [N+1] - [N] \tag{2.14a}$$

$$[N, a_q] = -a_q \qquad [N, a_q^+] = a_q^+ \tag{2.14b}$$

$$a_q = \frac{a\sqrt{[n]}}{\sqrt{N}} = \frac{\sqrt{[N+1]}a}{\sqrt{N+1}} \tag{2.14c}$$

$$a_q^+ = \frac{\sqrt{[N]}a^+}{\sqrt{N}} = \frac{a^+\sqrt{[N+1]}}{\sqrt{N+1}} \tag{2.14d}$$

while f_q and f_q^+ are the q-deformed fermionic annihilation and creation operators, respectively, obeying the relations [15]

$$f_q f_q^+ + q^{1/2} f_q^+ f_q = q^{-M/2} \tag{2.15a}$$

$$[M, f_q^+] = f_q^+ \qquad [M, f_q] = -f_q. \tag{2.15b}$$

In the above equations, N and M are the bosonic and fermionic number operators, respectively. It was shown that for $0 < q < 1$ any number of q-fermions can occupy a given state, in contrast to the case for ordinary fermions [15]. It was further shown that when $q = 1$, the nilpotency relations $f_{q-1}^2 = 0$, $(f_{q-1}^+)^2 = 0$ are realized in the weak sense, i.e. $f^2\|n\rangle_F = 0$, $(f^+)^2\|n\rangle_F = 0$, where $\|n\rangle_F$ spans the fermion Fock space, thereby reducing to the usual fermion operators when $q = 1$. But for $n > 1$, $0 < q < 1$, the equations $(f_q)^n \neq 0$, $(f_q^+)^n \neq 0$ hold.

3. *q*-supercoherent states and their properties

Let us denote the q-eigenstates of the q-deformed Hamiltonian H_q by $\|n,m\rangle$, where n and m are the boson and fermion numbers, respectively (note that m may be equal to any positive integer).

For convenience, one can define in equations (2.15) the following transformations:

$$f_q = q^{-M/4}F \qquad f_q^+ = F^+q^{-M/4}. \tag{3.1}$$

The basic anticommutation relation (2.15) now becomes

$$FF^{+} + qF^{+}F = 1 \tag{3.2a}$$

$$[M, F^{+}] = F^{+} \qquad [M, F] = -F. \tag{3.2b}$$

We now define three q-numbers as

$$[n]_{\mathrm{B}} = (q^{n} - q^{-n})/(q - q^{-1}) \tag{3.3a}$$

$$[n]_{\mathrm{F}} = (1 - (-q)^{n})/[1 - (-q)] \tag{3.3b}$$

$$[n]_{\mathrm{G}} = [(q^{-1/2})^{n} - (-q^{1/2})^{n}]/[q^{-1/2} - (-q^{1/2})]. \tag{3.3c}$$

Iterating equation (3.2a), we arrive at the formula

$$F(F^{+})^{n} - (-q)^{n}(F^{+})^{n}F = [n]_{\mathrm{F}}(F^{+})^{n-1}. \tag{3.3d}$$

In order to let $[n]_{\mathrm{F}}$ be positive and to be able to construct a super-Fock space based on the vacuum $\|0\rangle$, $a_q\|0\rangle = 0$, $F\|0\rangle = 0$, for these oscillators, one can easily show that q in equation (3.2a) must be taken to be real and in the range $0 < q < 1$.

Defining the vacuum state $\|0\rangle$ by the expression

$$\|0\rangle = \|0\rangle_{\mathrm{B}} \odot \|0\rangle_{\mathrm{F}} \tag{3.4}$$

where $\|0\rangle_{\mathrm{B}}$ and $\|0\rangle_{\mathrm{F}}$ are the vacuum states of the bosonic and fermionic operators, respectively, which obey

$$a_q\|0\rangle_{\mathrm{B}} = 0 \qquad F\|0\rangle_{\mathrm{F}} = 0 \tag{3.5}$$

we can construct the normalized n q-bosons and m q-fermions state by

$$\|n, m\rangle_q = ([n]_{\mathrm{B}}![m]_{\mathrm{F}}!)_{-1/2}(a_q^{+})^{n}(F^{+})^{m}\|0\rangle \tag{3.6}$$

where

$$[n]_{\mathrm{B}}! = [n]_{\mathrm{B}}[n-1]_{\mathrm{B}}\ldots[1]_{\mathrm{B}} \tag{3.7a}$$

$$[n]_{\mathrm{F}}! = [n]_{\mathrm{F}}[n-1]_{\mathrm{F}}\ldots[1]_{\mathrm{F}}. \tag{3.7b}$$

From equations (3.3d) and (3.6), the action of the q-boson operators a_q, a_q^{+} and the q-fermion operators F, F^{+} on the basis vectors can be given by

$$a_q\|n, m\rangle_q = \sqrt{[n]_{\mathrm{B}}}\|n-1, m\rangle_q \tag{3.8a}$$

$$a_q^{+}\|n, m\rangle_q = \sqrt{[n+1]_{\mathrm{B}}}\|n+1, m\rangle_q \tag{3.8b}$$

$$F\|n, m\rangle_q = \sqrt{[m]_{\mathrm{F}}}\|n, m-1\rangle_q \tag{3.8c}$$

$$F^{+}\|n, m\rangle_q = \sqrt{[m+1]_{\mathrm{F}}}\|n, m+1\rangle_q. \tag{3.8d}$$

From equation (3.3), and making use of an induction method, one can get the orthogonality relation [10]

$$ {}_q\langle n, m \| n', m' \rangle_q = \delta_{nn'}\delta_{mm'}. \tag{3.9}$$

Now we point out again that the q-fermion operators F and F^+ are not nilpotency operators. However, it can be seen from equations (3.6) and (3.8b) that when $q \to 1$,

$$(a_q^+)^n (F^+)^m \|0\rangle = \sqrt{[n]_\mathrm{B}![m]_\mathrm{F}!}\|n, m\rangle_q = 0 \qquad \text{for } m > 1$$

i.e. they are consistent with the usual fermion operators.

By making use of equations (2.13), (3.1) and (3.8), the energy level, corresponding to the state $\|n, m\rangle_q$, of this system can be expressed by

$$E_{n,m} = (\omega/2)\{([n]_\mathrm{B} + [n+1]_\mathrm{B}) + (q^{1/2}[m]_\mathrm{F} - [m+1]_\mathrm{F})q^{-m/2}\}. \tag{3.10}$$

In order to construct the q-supercoherent states (q-SCSs) of the supersymmetric quantum harmonic oscillators (SQHOs), we will find it useful to introduce the q-exponential functions for the q-bosonic operators and the q-fermionic operators, respectively, which are defined by the following expressions:

$$\exp_{\mathrm{B}q}(za_q^+) = \sum_{n=0}^{\infty} z^n (a_q^+)^n/[n]_\mathrm{B}! \tag{3.11a}$$

$$\exp_{\mathrm{FG}q}(\Psi_q F^+) = \sum_{m=0}^{\infty} (\Psi_q F^+)^m/([m]_\mathrm{F}![m]_\mathrm{G}!)^{1/2}. \tag{3.11b}$$

These q-exponential functions are the q-analogues of the classical ones. In equations (3.11a) and (3.11b), z is taken as the complex number and Ψ_q (or $\overline{\Psi_q}$) is taken as the pseudo-Grassmann variable. By this we mean that $\Psi_q\overline{\Psi_q} + \overline{\Psi_q}\Psi_q = 0$; but Ψ_q and $\overline{\Psi_q}$ are not nilpotent, $(\Psi_q)^n \neq 0$, $(\overline{\Psi_q})^n \neq 0$.

As for the usual Grassmann variables, Ψ_q and $\overline{\Psi_q}$ are taken to anticommute with F and F^+; but commute with the fermion number operator M, and when $q \to 1$, $\Psi_q \to \Psi$, $\overline{\Psi_q} \to \bar{\Psi}$, $(\Psi)^2 = 0$, $(\bar{\Psi})^2 = 0$.

We now construct q-SCSs. The q-SCSs of the SQHOs can be defined as follows:

$$\|z, \Psi_q\rangle = N(\bar{z}z, \overline{\Psi_q}\Psi_q) \exp_{\mathrm{B}q}(za_q^+) \exp_{\mathrm{FG}q}(-\Psi_q F^+)\|0\rangle \tag{3.12}$$

where $N(\bar{z}z, \overline{\Psi_q}\Psi_q)$ is the normalization factor. Since z is a complex variable and Ψ_q a pseudo-Grassmann variable, $\|z, \Psi_q\rangle$ are called q-SCSs.

By means of equations (3.11), the q-SCSs (3.12) can be rewritten as

$$\begin{aligned}\|z, \Psi_q\rangle &= N(\bar{z}z, \overline{\Psi_q}\Psi_q) \sum_{n,m=0}^{\infty} \frac{z^n (a_q^+)^n (-\Psi_q F^+)^m}{[n]_\mathrm{B}!([m]_\mathrm{F}![m]_\mathrm{G}!)^{1/2}}\|0\rangle \\ &= N(\bar{z}z, \overline{\Psi_q}\Psi_q) \sum_{n,m=0}^{\infty} \frac{(-1)^{(m/2)+m} z^n (\Psi_q)^m}{\sqrt{[n]_\mathrm{B}![m]_\mathrm{G}!}}\|n, m\rangle_q \end{aligned} \tag{3.13}$$

where $(m/2)$ stands for the integer part of $m/2$. In the above equation, we have used equation (3.6).

The q-SCSs of the q-SQHOs should be normalized in the form

$$\langle z, \Psi_q \| z, \Psi_q \rangle = 1. \tag{3.14}$$

Substituting equation (3.13) into equation (3.14) and making use of equation (3.9), one can obtain the following relation:

$$\langle z, \Psi_q \| z, \Psi_q \rangle = N^2(\bar{z}z, \overline{\Psi_q}\Psi_q) \exp_{\mathrm{B}q}(\bar{z}z) \exp_{\mathrm{G}q}(\overline{\Psi_q}\Psi_q) = 1 \tag{3.15a}$$

where

$$\exp_{\mathrm{G}q}(x) = \sum_{n=0}^{\infty} x^n/[n]_{\mathrm{G}}!. \tag{3.15b}$$

Then, the normalization factor is given by

$$N(\bar{z}z, \overline{\Psi_q}\Psi_q) = [\exp_{\mathrm{B}q}(\bar{z}z) \exp_{\mathrm{G}q}(\overline{\Psi_q}\Psi_q)]^{-1/2}. \tag{3.16}$$

The overlap of two q-SCSs is written as

$$\langle z', \Psi'_q \| z, \Psi_q \rangle = N(\bar{z}'z', \bar{\Psi}'_q\Psi'_q) N(\bar{z}z, \overline{\Psi_q}\Psi_q) \exp_{\mathrm{B}q}(\bar{z}'z) \exp_{\mathrm{G}q}(\bar{\Psi}'_q\Psi_q). \tag{3.17}$$

This means that the q-SCSs of the q-deformed SQHOs are not orthogonal to each other. They are linearly dependent.

From equation (3.13), using equations (3.8a) and (3.8c) and the anticommuting properties of Ψ_q, it is easily shown that

$$a_q \| z, \Psi_q \rangle = z \| z, \Psi_q \rangle \tag{3.18a}$$

$$F \| z, \Psi_q \rangle = \Psi_q \| z, q\Psi_q \rangle \tag{3.18b}$$

that is, the q-SCSs $\| z, \Psi_q \rangle$ are the eigenstates of a_q and F-operators.

The resolution of unity in the super-Fock space spanned by the base vectors from equation (3.6) is given by

$$I = \sum_{n,m=0}^{\infty} \| n, m \rangle_q {}_q\langle n, m \|. \tag{3.19}$$

As is well known, the core of coherent states is their completeness relation. In the present case, it can be shown that the q-SCSs defined by equation (3.12) form a complete set. And their completeness relation takes the form

$$\iint \mathrm{d}_q^2 z \, \mathrm{d}_{\mathrm{G}q}^2 \Psi_q \, \sigma(\bar{z}z, \overline{\Psi_q}\Psi_q) \| z, \Psi_q \rangle \langle z, \Psi_q \| = I \tag{3.20}$$

where

$$\begin{aligned} &\mathrm{d}_q^2 z = r \, \mathrm{d}_q r \, \mathrm{d}\varphi \qquad z = r\mathrm{e}^{\mathrm{i}\varphi} \qquad \mathrm{d}_{\mathrm{G}q}^2 \Psi_q = \mathrm{d}_{\mathrm{G}q}(\zeta\bar{\zeta}) \, \mathrm{d}\theta \\ &\overline{\Psi_q} = \bar{\zeta}\mathrm{e}^{-\mathrm{i}\theta} \qquad \Psi_q = \zeta\mathrm{e}^{\mathrm{i}\theta} \qquad \bar{\zeta}\zeta + \zeta\bar{\zeta} = 0. \end{aligned} \tag{3.21}$$

Note that the integral over $d_q r$ and $d_{Gq}(\zeta\bar{\zeta})$ is the q-integration while the integral over φ and θ is the usual integration. The weight q-superfunction $\sigma(\bar{z}z, \overline{\Psi_q}\Psi_q)$ in equation (3.20) is given by

$$\sigma(\bar{z}z, \overline{\Psi_q}\Psi_q) = [[2]_B/(2\pi)^2][\exp_{Bq}(\bar{z}z)\exp_{Gq}(\overline{\Psi_q}\Psi_q)]^{-1}[\exp_{Bq}(-\bar{z}z)\exp_{Gq}(\overline{\Psi_q}\Psi_q)]. \tag{3.22}$$

We now prove the completeness relation (3.20). Let us denote the LHS of equation (3.20) by L. Substituting equations (3.13), (3.16) and (3.22) into equation (3.20), we can obtain

$$\begin{aligned} L = & \sum_{n,m=0}^{\infty}\sum_{n',m'=0}^{\infty}[[2]_B/(2\pi)^2]\iint d_q^2 z\, d_{Gq}^2\Psi_q \exp_{Bq}(-\bar{z}z) \\ & \times \exp_{Gq}(\overline{\Psi_q}\Psi_q)\frac{(-1)^{(m/2)+(m'/2)+m+m'}\bar{z}^{n'}z^n(\Psi_q)^m(\overline{\Psi_q})^{m'}}{\sqrt{[n']_B![n]_B![m']_G![m]_G!}}\|n,m\rangle_{q\,q}\langle n',m'\| \\ = & \sum_{n,m=0}^{\infty}\sum_{n',m'=0}^{\infty}[[2]_B/(2\pi)^2]\iint r\, d_q r\, d\varphi\, d_{Gq}(\bar{\zeta}\zeta)\, d\theta \exp_{Bq}(-r^2) \\ & \times \exp_{Gq}(\bar{\zeta}\zeta)\frac{(-1)^{(m'/2)+(m/2)+m+m'}r^{n+n'}e^{i(n-n')\varphi}}{\sqrt{[n']_B![n]_B![m']_G![m]_G!}} \\ & \times (\zeta)^m(\bar{\zeta})^{m'}e^{i(m-m')\theta}\|n,m\rangle_{q\,q}\langle n',m'\|. \end{aligned} \tag{3.23}$$

By making use of the integration

$$\int_0^{2\pi} e^{i(n-m)\varphi}\, d\varphi = 2\pi\delta_{nm} \tag{3.24}$$

and integrating over φ and θ, equation (3.23) can be rewritten as

$$L = \sum_{n,m=0}^{\infty}[2]_B\iint r\, d_q r\, d_{Gq}(\zeta\bar{\zeta})\exp_{Bq}(-r^2)\exp_{Gq}(-\zeta\bar{\zeta})\frac{r^{2n}(\zeta)^m(\bar{\zeta})^m}{[n]_B![m]_G!}\|n,m\rangle_{q\,q}\langle n,m\|. \tag{3.25}$$

From the q-Euler formula for the function $\Gamma(x)$ [8]

$$\int_0^{\infty} x^n \exp_{Bq}(-x)\, d_q x = [n]_B! \tag{3.26}$$

and $r d_q r = d_q r^2/[2]_B$, it follows that

$$L = \sum_{n,m=0}^{\infty}\int d_{Gq}(\zeta\bar{\zeta})\frac{(\zeta\bar{\zeta})^m\exp_{Gq}(-\zeta\bar{\zeta})}{[m]_G!}\|n,m\rangle_{q\,q}\langle n,m\|. \tag{3.27}$$

For quantum groups, the Gq derivative in the above equation can be defined as

$$\frac{d f(x)}{d_{Gq}x} = \frac{f(x)-f(-qx)}{x-(-qx)}. \tag{3.28}$$

Hence, we arrive at the formula

$$\int_0^\infty \mathrm{d}_{\mathrm{G}q}x\, x^m \exp_{\mathrm{G}q}(-x) = [m]_{\mathrm{G}}!. \tag{3.29}$$

By means of equation (3.29), it follows that

$$L = \sum_{n,m=0}^{\infty} \|n, m\rangle_{q\,q}\langle n, m\| = I. \tag{3.30}$$

Thus, we have proven that the q-SCSs $\|z, \Psi_q\rangle$ defined by equation (3.13) are completeness and equation (3.20) holds.

With the aid of the completeness relation of the q-SCSs, one can expand an arbitrary vector $\|f\rangle$ as

$$\|f\rangle = \iint \mathrm{d}_q^2 z\, \mathrm{d}_{\mathrm{G}q}^2 \Psi_q\, \sigma(\bar{z}z, \overline{\Psi_q}\Psi_q)\|z, \Psi_q\rangle\langle z, \Psi_q\|f\rangle. \tag{3.31}$$

Setting $\|f\rangle = \|z', \Psi_q'\rangle$, an arbitrary q-SCS of the q-SQHOs, we then have

$$\|z'\Psi_q'\rangle = \iint \mathrm{d}_q^2 z\, \mathrm{d}_{\mathrm{G}q}^2 \Psi_q\, \sigma(\bar{z}z, \overline{\Psi_q}\Psi_q)\|z, \Psi_q\rangle\langle z, \Psi_q\|z', \Psi_q'\rangle. \tag{3.32}$$

This means that the system of q-supercoherent states is actually overcompleteness since it contains subsystems of q-SCSs which are complete systems.

4. Uncertainty relations

It is well known that the Heisenberg uncertainty principle is one of the most fundamental in quantum mechanics. We now study the q-superoscillators uncertainty relation for the q-SCSs $\|z, \Psi_q\rangle$ given by equation (3.12).

From equation (2.6), one can get the q-deformation formula for the operators a and a^+,

$$a_q = (2\omega)^{-1/2}(\omega Q_q + \mathrm{i}P_q) \tag{4.1a}$$

$$a_q^+ = (2\omega)^{-1/2}(\omega Q_q - \mathrm{i}P_q) \tag{4.1b}$$

where Q_q and P_q are the q-deformations of x and p_x, respectively. From equations (4.1), it follows that

$$Q_q = (2\omega)^{-1/2}(a_q + a_q^+) \tag{4.2a}$$

$$P_q = (2\omega)^{1/2}(a_q - a_q^+)/2\mathrm{i}. \tag{4.2b}$$

Now we can discuss the uncertainty relation for the q-SCSs. From equations (3.18a) and (4.2), it is straightforward to obtain

$$\langle Q_q\rangle \equiv \langle z, \Psi_q\|Q_q\|z, \Psi_q\rangle = (2\omega)^{-1/2}(\bar{z} + z) \tag{4.3a}$$

$$\langle Q_q^2\rangle \equiv \langle z, \Psi_q\|Q_q^2\|z, \Psi_q\rangle = (\exp_{\mathrm{B}q}(\bar{z}z))^{-1}[q\exp_{\mathrm{B}q}(q\bar{z}z) + \exp_{\mathrm{B}q}(q^{-1}\bar{z}z)] \times [2\omega(q+1)]^{-1} + (z^2 + 2\bar{z}z + \bar{z}^2)/2\omega \tag{4.3b}$$

$$\langle P_q\rangle \equiv \langle z, \Psi_q\|P_q\|z, \Psi_q\rangle = (2\omega)^{1/2}(z - \bar{z})/2\mathrm{i} \tag{4.4a}$$

$$\langle P_q^2\rangle \equiv \langle z, \Psi_q\|P_q^2\|z, \Psi_q\rangle = -a^2/(q+1)[\exp_{\mathrm{B}q}(\bar{z}z)]^{-1}[q\exp_{\mathrm{B}q}(q\bar{z}z) + \exp_{\mathrm{B}q}(q^{-1}\bar{z}z)] + a^2(z - \bar{z})^2 \tag{4.4b}$$

where $a = \sqrt{2\omega}/2\mathrm{i}$. From equations (4.3) and (4.4), it is easy to arrive at the formulae

$$\Delta Q_q^2 \equiv \langle Q_q^2\rangle - \langle Q_q\rangle^2 = [\exp_{\mathrm{B}q}(\bar{z}z)]^{-1}[q\exp_{\mathrm{B}q}(q\bar{z}z) + \exp_{\mathrm{B}q}(q^{-1}\bar{z}z)]/2\omega(q+1) \quad (4.5a)$$

$$\Delta P_q^2 \equiv \langle P_q^2\rangle - \langle P_q\rangle^2 = [\exp_{\mathrm{B}q}(\bar{z}z)]^{-1}[q\exp_{\mathrm{B}q}(q\bar{z}z) + \exp_{\mathrm{B}q}(q^{-1}\bar{z}z)]\omega/2(q+1). \quad (4.5b)$$

Thus, we have

$$\Delta Q_q^2, \Delta P_q^2 = [\exp_{\mathrm{B}q}(\bar{z}z)]^{-2}[q\exp_{\mathrm{B}q}(q\bar{z}z) + \exp_{\mathrm{B}q}(q^{-1}\bar{z}z)]^2/4(q+1)^2. \quad (4.6)$$

On the other hand, we note that the commutation relation of the operators Q_q and P_q defined by equations (4.1) is

$$[Q_q, P_q] = \mathrm{i}[a_q, a_q^+] = \mathrm{i}([N+1]_{\mathrm{B}} - [N]_{\mathrm{B}}). \quad (4.7)$$

Hence, the q-SCSs of the q-superoscillators give the commutator $[Q_q, P_q]$ the following result

$$\begin{aligned}\langle[Q_q, P_q]\rangle &\equiv \langle z, \Psi_q\|[Q_q, P_q]\|z, \Psi_q\rangle \\ &= \mathrm{i}(\exp_{\mathrm{B}q}(\bar{z}z))^{-1}(q\exp_{\mathrm{B}q}(q\bar{z}z) + \exp_{\mathrm{B}q}(q^{-1}\bar{z}z))/(q+1) \end{aligned} \quad (4.8)$$

therefore, the uncertainty relation for the q-SCSs $\|z, \Psi_q\rangle$ is given by

$$\Delta Q_q^2, \Delta P_q^2 = \|\langle[Q_q, P_q]\rangle\|^2/4. \quad (4.9)$$

This means that the q-deformed canonical coordinates and canonical momenta satisfy the minimum uncertainty relations. The result of equation (4.9) shows that the q-SCSs $\|z, \Psi_q\rangle$ are the minimum uncertainty states [1].

5. Concluding remarks

We have obtained the solution of the quantum harmonic oscillators in the supersymmetric quantum systems and introduced their q-deformations. We have also constructed the q-SCS of the q-superoscillators, discussed their orthogonality and completeness relations, and investigated the uncertainty relation for the q-SCSs. We have found that the q-SCSs are the minimum uncertainty states of the superoscillators for all values of z and $0 \leqslant q \leqslant 1$ [11].

It is possible to generalize this kind of q-SCS to other supersymmetric quantum systems, e.g. the q-JC model. It is also interesting to discuss the q-SCS path integral representation of this model. We will give the results elsewhere.

Acknowledgments

The research was supported by the Foundation of the Science and Technology Committee of Hunan.

References

[1] Klauder R J and Skagerstam B S 1980 *Coherent States* (Singapore: World Scientific)
[2] Quesne C 1990 *J. Phys. A: Math. Gen.* **23** 5383, 5411
[3] Schmitt and Muftri 1990 *J. Phys. A: Math. Gen.* **23** L861
[4] Bano L and Rowe 1990 *J. Math. Phys.* **31** 14
[5] Drinfeld V G 1986 Quantum groups *Proc. I C M Berkeley* 789
[6] Jinbo M 1985 *Lett. Math. Phys.* **10** 63; 1986 *Lett. Math. Phys.* **11** 247
[7] Biedenham L C 1989 *J. Phys. A: Math. Gen.* **22** L873
[8] Gray R W and Nelson C A 1990 *J. Phys. A: Math. Gen.* **23** L945
[9] Bracken A J, McAnally D S, Zhang R B and Gould M D 1991 *J. Phys. A: Math. Gen.* **24** 1379
[10] Hao S R 1993 *Acta Phys. Sinica* **42** 691
[11] Witten E 1981 *Nucl. Phys.* B **188** 513
[12] Beckers J and Debergh N 1992 *Phys. Lett.* **286B** 290
[13] Bagchi B 1993 *Phys. Lett.* **309B** 85
[14] Chang Z, Chen W and Guo H Y 1990 *J. Phys. A: Math. Gen.* **23** 4185
[15] Parthasathy R and Viswantathan K S 1991 *J. Phys. A: Math. Gen.* **24** 613